Intermediate test Waves and Optics - 9 December 2013

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This test contains 4 questions on 3 pages.

A few preliminary remarks:

- Answers may be given in Dutch.
- Please answer questions 3 & 4 on another (double) sheet of paper than questions 1 & 2.
- Put your name and student number at the top of all sheets.
- Put your student card at the edge of the desk for checking by the assistants and show it when handing in your test.

Question 1 (8 points): plane harmonic waves

A plane harmonic electromagnetic wave is specified (in SI units) by the following wave function:

$$\vec{\mathbf{E}} = \left(4\hat{\mathbf{i}} - 6\hat{\mathbf{j}}\right)\left(10^3\ \mathrm{V/m}\right)\ \cos\left[\left(3x+2y\right)\pi\times10^7 - 1.26\times10^{16}t\right]$$

$k = \frac{2\pi}{\lambda}.$

with $\hat{\mathbf{i}}, \hat{\mathbf{j}}$ the unit vectors along the x- and y-axis, respectively.

$\omega = kv.$

Questions:

$\omega = 2\pi \frac{v}{\lambda} = 2\pi f$

a) Draw the direction in the x-y plane along which the electric field oscillates.

b) What is the scalar value of the amplitude of the electric field ?

c) What is the direction of propagation of the wave ? Indicate this direction in the drawing used in the answer of question a).

d) What is the wavelength of the wave ?

e) What is the frequency of the wave ?

Add the units to the numbers calculated.

(Question 2 on the next page)

Question 2 (7 points): Fresnel equations, transmittance

A beam of light in air strikes the surface of a smooth piece of plastic (with index of refraction 1.5) at an angle of incidence of 30 degrees. The incident light has an electric field component parallel to the plane of incidence with an amplitude of 10.0 V/m and a component perpendicular to the plane of incidence with an amplitude of 20 V/m.

a) Calculate the **amplitude** of the **parallel** component of the electric field of the **reflected** beam.

b) Calculate the **perpendicular** component of the **transmittance**.

Make use of the appropriate Fresnel equations given below.

$$r_\perp = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t} \quad (4.34)$$

$$\boxed{\frac{I_r}{I_i} = r^2}$$

$$t_\perp = \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t} \quad (4.35)$$

$$r_\parallel = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_i \cos\theta_t + n_t \cos\theta_i} \quad (4.40)$$

$$t_\parallel = \frac{2n_i \cos\theta_i}{n_i \cos\theta_t + n_t \cos\theta_i} \quad (4.41)$$

For question b), in case you need to derive the appropriate equation of the transmittance, the following expression for the irradiance I of a harmonic electromagnetic wave with amplitude E_0 may be useful:

$$I = \frac{c\,\varepsilon_0}{2} E_0^2$$

$I_r + I_t = I_i$

$\frac{I_r}{I_i} + \frac{I_t}{I_i} = 1$

$\frac{n \frac{c\varepsilon_0}{2} E_{0t}^2 \cdot \cos\theta_t}{\frac{c\varepsilon_0}{2} E_{0i}^2 \cos\theta_i}$

$\frac{V\varepsilon\mu}{2\mu} E_0^2$

$\frac{c}{V} = n$

$V = \frac{c}{n}$

$\frac{V}{2\mu V^2} = \frac{1}{2\mu V} = \frac{n}{2\mu c}$

$\frac{n\varepsilon_0}{2\mu\varepsilon_0 c} = \frac{nc^2\varepsilon_0}{2c} =$

(Question 3 on the next page)

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$\frac{n_t\cos\theta_t}{n_i\cos\theta_i}\frac{E_{0t}^2}{E_{0i}^2}$

Question 3 (7 points): spherical and cylindrical waves

The amplitude of a spherical wave emitted by a point source is proportional to $1/r^2$, with r the distance to the point source.

a) Show that this relationship follows from the law of conservation of energy.

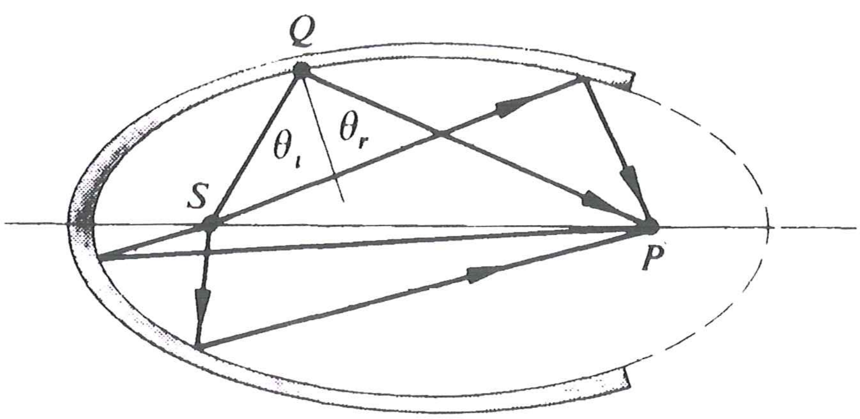
b) Use the same reasoning (thus using the law of conservation of energy) to deduce the relationship between the amplitude of a cylindrical wave and the distance r to the line source that generates the cylindrical wave.

Question 4 (8 points): Fermat's principle

Fermat's principle allows to determine the manner in which light propagates.

a) What does the **original formulation** of Fermat's principle say about the path light will follow ? Give **two** equivalent versions. (No derivations are asked for, just state the principle.)

b) In some situations, Fermat's principle is not valid. In the lectures, an example of such a situation was discussed involving a source S and observation point P in the foci of an ellipsoid. The figure below should bring this situation to mind. For an ellipsoid, all paths SQP (with Q any point on the ellipsoid) have the same length.



Use this example to explain a situation in which Fermat's principle is not valid.

c) The existence of situations in which Fermat's principle is not valid has resulted in a **modern formulation** of Fermat's principle which is always valid. Give this modern formulation and explain its meaning.